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Multiple Attribute Decision Making Problem Using Interval-Valued Picture Fuzzy Graphs

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Abstract

This study addresses challenges in multiple attribute decision-making (MADM) within the picture fuzzy domain, particularly the lack of consideration for relationships among attributes. To overcome this, we extend the concept to an interval-valued picture graph. We propose an advancedMADM method capable of effectively managing complex issues by capturing attribute connections that existing methods struggle with. The paper introduces interval-valued picture fuzzy graphs, explores their properties, and presents an algorithm based on interval-valued picture fuzzy graph to tackle these challenges. A numerical example and comparative analysis demonstrate the method's effectiveness and practicality.

Keywords: Picture fuzzy graph, interval-valued picture fuzzy graph, complete picture fuzzy graph, strong picture fuzzy graph, decision making problem, score value.

1|Introduction

Currently, graphs sometimes fail to fully represent systems due to uncertainty in the system's parameters. For example, a social network can be depicted as a graph where nodes represent accounts (such as institutions or individuals) and edges represent the connections between them. If the quality of these connections-whether good or bad-depends on the frequency of interactions, then fuzziness must be incorporated into the representation.

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Rosenfeld first introduced the concept of fuzzy graphs in 1975 by defining fuzzy relations on fuzzy sets. A Picture Fuzzy Set (PFS) is an extension of the Intuitionistic Fuzzy Set (IFS), offering greater precision, flexibility, and compatibility. The concept of PFS was introduced by Coung *et al.* [5] in 2013, adding a new component to IFS to account for neutral membership degrees. While IFS provides membership and non-membership degrees, PFS includes a positive membership degree, a neutral membership degree, and a negative membership degree, with the sum of these three degrees being ≤ 1. PFS models are particularly useful in situations involving multiple opinions, such as voting, where participants might choose to vote for, against, abstain, or refuse. PFS has various applications in fields like system analysis, operations research, economics, medicine, computer science, engineering, and mathematics. Interval-valued picture fuzzy det ($IVPFS$) is an extension of PFS having lot of applications in reality.

Review of literature. In 1965, Zadeh [14] introduced fuzzy sets to deal with this type of situation. The membership value of an element in a fuzzy set lies between 0 and 1. This describes more accurately the problems in our daily life. There are so many problems in which every member has a degree of belongingness and not belongingness. Atanassov [3] designs this type of problem into a set name as an intuitionistic fuzzy set. The concept of neutrality degree of belongingness extended the intuitionistic fuzzy set into a picture fuzzy set, proposed by Cuong and Kreinovich [5]. There are so many problems that are not described by an intuitionistic fuzzy set, for example, the election process in democratic nations. In an election station, it is found that there are 60 % ballots for a nominated candidate, 12% ballots canceled, 38 % ballots against the candidate and 7% electors bypass the election process. This type of situation can not be expressed by an intuitionistic fuzzy set but can be expressed by the concept of a picture fuzzy set. Pramanik *et al.* [8] have defined interval-valued fuzzy threshold graphs and discuss various properties. Pramanik *et al.* studied interval-valued fuzzy planar graph [9], interval valued fuzzy graph [10]. In 2013, Karunambigai *et al.* [7] studied balanced intuitionistic fuzzy graphs. In 2014, Akram *et al.* [1] defined intuitionistic fuzzy cycles, planar graphs, and tree [2]. In 2015, Zhao *et al.* [15] studied interval-valued neutrosophic sets and multiattribute decision-making based on generalized weighted aggregation operators. Recently, in 2019, Zuo *et al.* [16] proposed the definition of PFG based on PF-relation. Das *et al.* [6] have studied picture fuzzy threshold graphs. Regular PFGs have been studied by Xiao *et al.* [13]. In 2021, Amanathulla *et al.* [11] discussed multiple attribute decision-making methods in PF-environment. Khatun *et al.* have studied Picture fuzzy cubic graphs and their applications [17], m-polar picture fuzzy graph [18]. Adhikari *et al.* [4] have studied interval-valued picture fuzzy graph. Banerjee *et al.* [20] have introduced Optimization of disaster management using split domination in picture fuzzy graphs.

An advanced level research in various types of fuzzy dominated mapping is going on by Rasham *et al.* [21, 22, 23]. Several game theory problems has been solved by Seikh *et al.* [24] and Dutta *et al.* [25]. In 2024, Gu *et al.* [26] have studied research on improving the financial capacity of farmers based on fuzzy analytic hierarchy process. Recently, several researchers have studied efficient resource allocation management in multicommodity networks [27], an optimized model for neutrosophic multi-choice goal programming [28], A new interval for ranking alternatives in multi attribute decision making problems [29], Extended DEA method for solving multi-objective transportation problem with Fermatean fuzzy sets [30], A new multi-attribute decision-making method for interval data using support vector machine [31] and An extension of the TOPSIS for multi-attribute group decision making under neutrosophic environment [28]. Various types of decision making problems and real world problems has been studied in [33]-[45].

Motivation. The motivation for the proposed work stems from the growing complexity and interdependence of attributes in real-world decision-making scenarios. Traditional methods in $MADM$ often struggle to capture the nuanced relationships between attributes, particularly in environments characterized by uncertainty and ambiguity. $IVPFS$ offer a more refined approach to modeling uncertainty by considering not just positive and negative membership degrees, but also a neutral membership degree, providing a more comprehensive representation of real-world scenarios and also allow the flexibilities of the degree of memberships. However, existing $MADM$ methods do not fully leverage the potential of $IVPFS$ in representing and analyzing the intricate interconnections between attributes. This gap becomes especially critical in complex systems, such as social networks, financial systems, or engineering designs, where the interactions between different components or attributes significantly influence the overall outcome. To address this challenge, the paper proposes the use of interval-valued picture fuzzy graph as a novel tool for $MADM$, enabling the effective capture and analysis of attribute relationships. By integrating graph theory with $IVPFS$, this approach not only enhances the

decision-making process by accounting for the connections among attributes but also provides a more accurate and flexible framework for dealing with complex decision-making problems. This research aims to bridge the gap in existing $MADM$ methodologies and offer a robust solution that is better suited to handle the intricacies of real-world applications.

The remaining part of the article is arranged as follow:
• In Section 2, preliminaries are presented.
• PFG and $IVPFG$ have been studied in Section 3.
• Decision-making algorithms based on $MADM$ for complicated problems are given in Section 4.
• In Section 5, a numerical example of $MADM$ technique with $IVPF$-domain is used to present the applications of the proposed decision-making technique, the comparative analysis of the proposed decision-making method is also given in this section.
• Conclusion and future works have been presented in the last section.

2|Preliminaries

A PFS is an extension of IFS and $IVPFS$ is an extension of PFS. The definition of PFS and $IVPFS$ are given below.

Definition 1. [5] *Let Y be an universal set and let $y \in Y$. A PFS B is characterized by a positive membership function (positive-mf) $POS_B(y)$, a neutral-mf $NEU_B(y)$, and a negative-mf $NEG_B(y)$, where $POS_B(y)$, $NEU_B(y)$, $NEG_B(y) \in [0,1]$.*
A PFS B can be expressed by $B = \{(y : POS_B(y), NEU_B(y), NEG_B(y)), y \in Y\}$.

Definition 2. [19] *Let Y be an universal set and the set of all closed subsets of $[0,1]$ be denoted by $I[0,1]$. Then the $IVPFS$ is defined by*
$B = \{(y, POS_B(y), NEU_B(y), NEG_B(y)) : y \in Y\}$, where $POS_B : Y \to I[0,1]$, $NEU_B : Y \to I[0,1]$, $NEG_B : Y \to I[0,1]$ and for all $y \in Y$, $0 \le supPOS_B(y) + supNEU_B(y) + supNEG_B(y) \le 1$. The intervals $POS_B(y), NEU_B(y), NEG_B(y)$ are the positive membership degree, neutral membership degree $NEU_B(y)$, and negative membership degree of y to B respectively.

For example, if $POS_B(y) = [POS^l_B(y), POS^u_B(y)]$, $NEU_B(y) = [NEU^l_B(y), NEU^u_B(y)]$ and $NEG_B(y) = [NEG^l_B(y), NEG^u_B(y)]$,then
$B = \{(y, [POS^l_B(y), POS^u_B(y)], [NEU^l_B(y), NEU^u_B(y)], [NEG^l_B(y), NEG^u_B(y)]) : y \in Y\}$, where $0 \le supPOS^u_B(y) + supNEU^u_B(y) + supNEG^u_B(y) \le 1$ for all $y \in Y$.

Definition 3. [19] *Let $S = (POS_S, NEU_S, NEG_S)$ and $T = (POS_T, NEU_T, NEG_T)$ be PFS's on the set Y. Then if $S = (POS_S, NEU_S, NEG_S)$ is a picture fuzzy relation on Y then $S = (POS_S, NEU_S, NEG_S)$ is a picture fuzzy relation on $T = (POS_T, NEU_T, NEG_T)$*
if $POS_T(\xi, \eta) \le POS_S(\xi) \wedge POS_S(\eta)$
$NEU_T(\xi, \eta) \ge NEU_S(\xi) \vee NEU_S(\eta)$
$NEG_T(\xi, \eta) \ge NEG_S(\xi) \vee NEG_S(\eta)$

A picture fuzzy relation S on Y is said to be symmetric if $POS_S(\xi, \eta) = POS_S(\eta, \xi)$, $NEU_S(\xi, \eta) = NEU_S(\eta, \xi)$, $NEG_S(\xi, \eta) = NEG_S(\eta, \xi)$ $POS_T(\xi, \eta) = POS_T(\eta, \xi)$, $NEU_T(\xi, \eta) = NEU_T(\eta, \xi)$, $NEG_T(\xi, \eta) = NEG_T(\eta, \xi)$ for all $\xi, \eta \in Y$.

Definition 4. [11] *Let $f_n = (\alpha, \beta, \gamma)$ be a picture fuzzy number, then the score function of f_n is denoted by $scor(f_n)$ and is defined by $scor(f_n) = \frac{1+\alpha-2\beta-\gamma}{2}$.*

Observation 1. *Let f_1 and f_2 be two picture fuzzy numbers then $scor(f_1) > scor(f_2) \Rightarrow f_1 > f_2$.*

Picture fuzzy graph and interval valued picture fuzzy graph

The definition, example, and some properties of PFG and $IVPFG$ have been presented in this section.

Definition 5. *Let $\beta = ([POS^l(y), POS^u(y)], [NEU^l(y), NEU^u(y)], [NEG^l(y), NEG^u(y)])$ be an interval PF number, then the score of β is defined by*
$score(\beta) = \frac{1}{4}[2 + POS^l + POS^u - 2NEU^l - 2NEU^u - NEG^l - NEG^u]$.

Observation 2. *Let β_1 and β_2 be two interval PF-numbers then score$(\beta_1) >$ score$(\beta_2) \Rightarrow \beta_1 \Rightarrow \beta_2$.*

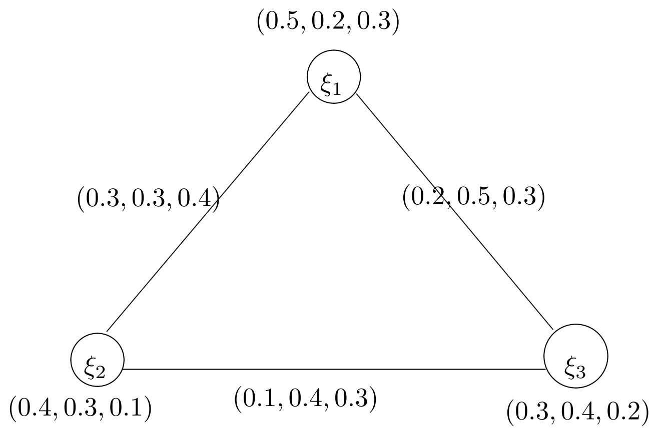

FIGURE 1. A PFG

Definition 6. *A PFG with underlying set V is defined by $G = (S, T)$, where, (i) The functions $POS_S : V \to [0, 1]$, $NEU_S : V \to [0, 1]$, $NEG_S : V \to [0, 1]$ denote the positive, neutral and negative membership value of $\xi_i \in V$ respectively and $0 \le POS_S(\xi_k) + NEU_S(\xi_k) + NEG_S(\xi_k) \le 1$ for all $\xi_k \in V$, $k = 1, 2, \ldots, n$.*
(ii) The functions $POS_T : E \to [0, 1]$, $NEU_T : E \to [0, 1]$, $NEG_T : E \to [0, 1]$ (where $E \subseteq V \times V$) are defined by
$POS_T(\xi_k, \xi_l) \le POS_S(\xi_k) \wedge POS_S(\xi_l)$
$NEU_T(\xi_k, \xi_l) \ge NEU_S(\xi_k) \vee NEU_S(\xi_l)$
$NEG_T(\xi_k, \xi_l) \ge NEG_S(\xi_k) \vee NEG_S(\xi_l)$ *denotes the positive, neutral and negative membership value of the edge $(\xi_k, \xi_l) \in E$ respectively, where, $0 \le POS_T(\xi_k, \xi_l) + NEU_T(\xi_k, \xi_l) + NEG_T(\xi_k, \xi_l) \le 1$ for all $(\xi_k, \xi_l) \in E$ and $k, l = 1, 2, \ldots, n$.*

Here, S is the picture fuzzy vertex set of V and T is a picture fuzzy edge set of E. It is to be noted that T is a symmetric picture fuzzy relation on S. A PFG is shown in Fig. 1.

Definition 7. *A PFG $G = (S, T)$ is said to be complete if*
$POS_T(\xi_k, \xi_l) = POS_S(\xi_k) \wedge POS_S(\xi_l)$
$NEU_T(\xi_k, \xi_l) = NEU_S(\xi_k) \vee NEU_S(\xi_l)$
$NEG_T(\xi_k, \xi_l) = NEG_S(\xi_k) \vee NEG_S(\xi_l)$ *for all $\xi_k, \xi_l \in V$.*

A complete PFG is shown in Fig. 2.

Definition 8. *A PFG $G = (S, T)$ is called strong PFG if*
$POS_T(\xi_k, \xi_l) = POS_S(\xi_k) \wedge POS_S(\xi_l)$
$NEU_T(\xi_k, \xi_l) = NEU_S(\xi_k) \vee NEU_S(\xi_l)$
$NEG_T(\xi_k, \xi_l) = NEG_S(\xi_k) \vee NEG_S(\xi_l)$ *for all $(\xi_k, \xi_l) \in E$.*

A strong PFG is depicted in Fig. 3.

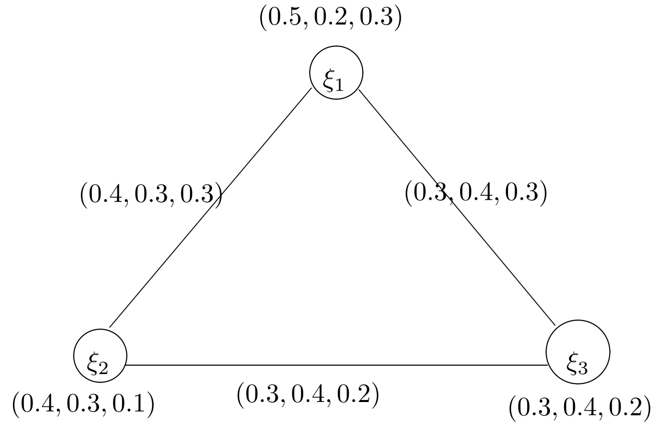

FIGURE 2. A complete PFG

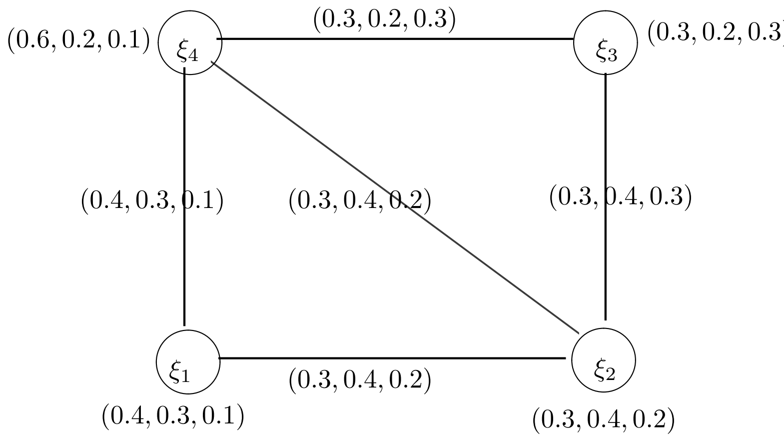

FIGURE 3. A strong PFG

Definition 9. *Let $G^* = (V, E)$ be a crisp graph. Then an $IVPFG$ on G^* is a pair $G = (S, T)$, where $S = ([POS_{SL}, POS_{SU}], [NEU_{SL}, NEU_{SU}], [NEG_{SL}, NEG_{SU}])$ is an $IVPFS$ on V and $T = ([POS_{TL}, POS_{TU}], [NEU_{TL}, NEU_{TU}], [NEG_{TL}, NEG_{TU}])$ is an $IVPF$-relation on E satisfying the following conditions:*

(i) $V = \{\xi_1, \xi_2, \dots, \xi_n\}$ so that
$POS_{SL} : V \to [0,1]$, $POS_{SU} : V \to [0,1]$
$NEU_{SL} : V \to [0,1]$, $NEU_{SU} : V \to [0,1]$
$NEG_{SL} : V \to [0,1]$, $NEG_{SU} : V \to [0,1]$ *respectively denote the positive, neutral, negative membership degree of any element $\eta \in V$ and $0 \le POS_S(\xi_i) + NEU_S(\xi_i) + NEG_S(\xi_i) \le 1$ for all $\xi_i \in V$, $i = 1, 2, \dots n$.*

(ii) The mappings $POS_{TL} : V \times V \to [0,1]$, $POS_{TU} : V \times V \to [0,1]$, $NEU_{TL} : V \times V \to [0,1]$, $NEU_{TU} : V \times V \to [0,1]$, $NEG_{TL} : V \times V \to [0,1]$, $POS_{TU} : V \times V \to [0,1]$ are such that

$POS_{TL}(\xi_i, \xi_j) \le POS_{SL}(\xi_i) \wedge POS_{SL}(\xi_j)$, $POS_{TU}(\xi_i, \xi_j) \le POS_{SU}(\xi_i) \wedge POS_{SU}(\xi_j)$, $NEU_{TL}(\xi_i, \xi_j) \ge NEU_{SL}(\xi_i) \vee NEU_{SL}(\xi_j)$, $NEU_{TU}(\xi_i, \xi_j) \ge NEU_{SU}(\xi_i) \vee NEU_{SU}(\xi_j)$, $NEG_{TL}(\xi_i, \xi_j) \ge NEG_{SL}(\xi_i) \vee NEG_{SL}(\xi_j)$, $NEU_{TU}(\xi_i, \xi_j) \ge NEG_{SU}(\xi_i) \vee NEG_{SU}(\xi_j)$ respectively denote +ve, neutral and negative membership value of the edges $(\xi_i, \xi_j) \in E$, where $0 \le POS_{TU}(\xi_i, \xi_j) + NEU_{TU}(\xi_i, \xi_j) + NEG_{TU}(\xi_i, \xi_j) \le 1$ for all $(\xi_i, \xi_j) \in E$, $i, j = 1, 2, \dots n$. Here, S is the interval valued picture fuzzy vertex set of V and T is the intervalued picture fuzzy edge set of E.

An $IVPFG$ is depected in Fig. 4. Let $G^* = (V, E)$, where $V = \{\xi_1, \xi_2, \xi_3, \xi_4\}$, and $E = \{\xi_1\xi_2, \xi_1\xi_3, \xi_2\xi_3, \xi_2\xi_4\}$. Let S and T be two PFSs of V and E respectively, defined below:

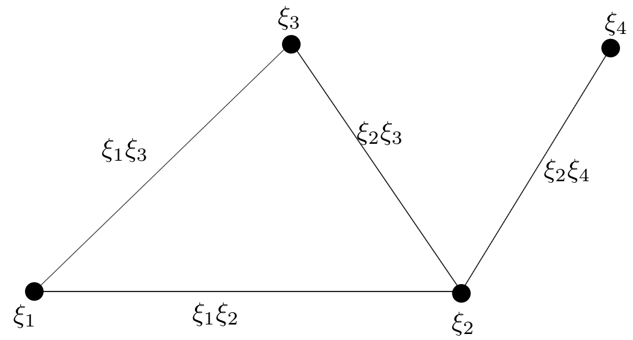

FIGURE 4. An $IVPFG$

	ξ_1	ξ_2	ξ_3	ξ_4
$[POS_{SL}, POS_{SU}]$	[0.03,0.13]	[0.1,0.17]	[0.03,0.06]	[0.17,0.2]
$[NEU_{SL}, NEU_{SU}]$	[0.07,0.2]	[0.07,0.17]	[0.0,0.01]	[0.06,0.13]
$[NEG_{SL}, NEG_{SU}]$	[0.1,0.23]	[0.07,0.17]	[0.03,0.06]	[0.03,0.07]

TABLE 1. Membership value of nodes in $IVPFG$

	$\xi_1\xi_2$	$\xi_1\xi_3$	$\xi_2\xi_3$	$\xi_2\xi_4$
$[POS_{TL}, POS_{TU}]$	[0.06,0.1]	[0.03,0.23]	[0.03,0.06]	[0.06,0.1]
$[NEU_{TL}, NEU_{TU}]$	[0.034,0.14]	[0.034,0.27]	[0.034,0.076]	[0.1,0.2]
$[NEG_{TL}, NEG_{TU}]$	[0.14,0.24]	[0.034,0.24]	[0.17,0.27]	[0.067,0.17]

TABLE 2. Membership value of edgess in $IVPFG$

In Fig. 4,
(i) $(\xi_1, [0.03, 0.13], [0.07, 0.2], [0.1, 0.23])$, $(\xi_2, [0.1, 0.17], [0.07, 0.17], [0.07, 0.17])$,
$(\xi_3, [0.03, 06], [0.0, 0.1], [0.03, 0.06])$,
$(\xi_4, [0.17, 0.2], [0.06, 0.13], [0.03, 0.07])$ are the interval valued picture fuzzy vertices.
(ii) $(\xi_1\xi_2, [0.06, 0.1], [0.034, 0.14], [0.14, 0.24])$, $(\xi_1\xi_3, [0.03, 0.23], [0.034, 0.27], [0.034, 0.24])$, $(\xi_2\xi_3, [0.03, 0.06], [0.034, 0.067], [0.17, 0.27])$, $(\xi_2\xi_4, [0.06, 0.1], [0.1, 0.2], [0.067, 0.17])$ are the picture fuzzy edges.
(iii) $(\xi_1, [0.03, 0.13], [0.07, 0.2], [0.1, 0.23])$ and $(\xi_2, [0.1, 017], [0.07, 0.17], [0.07, 0.17])$;
$(\xi_2, [0.1, 0.17], [0.07, 0.17], [0.07, 0.17])$ and $(\xi_4, [0.17, 0.2], [0.06, 0.13], [0.3, 0.07])$ etc. are picture fuzzy adjacent vertices.
(iii) $(\xi_1\xi_2, [0.06, 0.1], [0.034, 0.14]$ and $(\xi_1\xi_3, [0.03, 0.23], [0.034, 0.27], [0.034, 0.24])$;
$(\xi_1\xi_2, [0.06, 0.1], [0.034, 0.14]$ and $(\xi_2\xi_4, [0.06, 0.1], [0.1, 0.2], [0.067, 0.17])$ are the interval valued picture fuzzy adjacent edges.

The adjacency matrix corresponding to the $IVPFG$ of Fig. 4 is given by

$$M = \begin{bmatrix} ([0.1, 0.13], [0.07, 0.2], [0.1, 0.23]) & ([0.06, 0.1], [0.034, 0.14], [0.14, 0.24]) & ([0.03, 0.23], [0.034, 0.27], [0.034, 0.24]) & ([0.0, 0.0], [0.0, 0.0], [0.0, 0.0]) \\ ([0.06, 0.1], [0.034, 0.14], [0.14, 0.24]) & ([0.1, 0.17], [0.07, 0.17], [0.07, 0.17]) & ([0.03, 0.06], [0.034, 0.067], [0.17, 0.27]) & ([0.06, 0.1], [0.1, 0.2], [0.067, 0.17]) \\ ([0.03, 0.23], [0.034, 0.27], [0.034, 0.24]) & ([0.03, 0.06], [0.034, 0.067], [0.17, 0.27]) & ([0.03, 0.06], [0.0, 0.0.1], [0.03, 0.06]) & ([0.0, 0.0], [0.0, 0.0], [0.0, 0.0]) \\ ([0.0, 0.13], [0.0, 0.0], [0.0, 0.0]) & ([0.06, 0.1], [0.1, 0.2], [0.067, 0.17]) & ([0.0, 0.0], [0.0, 0.0], [0.0, 0.0]) & ([0.17, 0.2], [0.06, 0.13], [0.03, 0.07]) \end{bmatrix}$$

Definition 10. *Let $G = (R, S)$ be an $IVPFG$. Then the degree of a node v in $IVPFG$ is defined by*

$$d(v) = ([d_{POS_L}(v), d_{POS_U}(v)], [d_{NEU_L}(v), d_{NEU_U}(v)], [d_{NEG_L}(v), d_{NEG_U}(v)])$$

where,

$$d_{POS_L}(v) = \sum_{u \neq v} POS_{SL}(vu), d_{POS_U}(v) = \sum_{u \neq v} POS_{SU}(vu).$$
$$d_{NEU_L}(v) = \sum_{u \neq v} NEU_{SL}(vu), d_{NEU_U}(v) = \sum_{u \neq v} NEU_{SU}(vu).$$

$$d_{NEG_L}(v) = \sum_{u \neq v} NEG_{SL}(vu), d_{NEG_U}(v) = \sum_{u \neq v} NEG_{SU}(vu).$$

Definition 11. *Let $G = (R, S)$ be an $IVPFG$. Then the order of G is defined by*

$$O(G) = (O_{POS}(G), (O_{NEU}(G)), (O_{NEG}(G))$$

where,

$$O_{POS}(G) = [\sum_{u \in V} POS_{RL}(v), \sum_{u \in V} POS_{RU}(v)],$$
$$O_{NEU}(G) = [\sum_{u \in V} NEU_{RL}(v), \sum_{u \in V} NEU_{RU}(v)],$$
$$O_{NEG}(G) = [\sum_{u \in V} NEG_{RL}(v), \sum_{u \in V} NEG_{RU}(v)].$$

Definition 12. *Let $G = (R, S)$ be an $IVPFG$. Then the size of G is defined by*

$$S(G) = (S_{POS}(G), (S_{NEU}(G)), (S_{NEG}(G))$$

where,

$$S_{POS}(G) = [\sum_{u \neq v} POS_{SL}(uv), \sum_{u \neq v} POS_{SU}(uv)],$$
$$S_{NEU}(G) = [\sum_{u \neq v} NEU_{SL}(uv), \sum_{u \neq v} NEU_{SU}(uv)],$$
$$S_{NEG}(G) = [\sum_{u \neq v} NEG_{SL}(uv), \sum_{u \neq v} NEG_{SU}(uv)].$$

PFG based $MADM$ method. In 2021, Amanathulla *et al.* [11]. proposed an algorithm for multiple attribute decision making to find the optimal choice with picture fuzzy domain. The proposed algorithm is given below.

Algorithm 1

Step 1: Compute the coefficient of impact between the attributes C_r and C_s by $\delta_{rs} = \frac{POS_{rs} + (1 - NEU_{rs})(1 - NEG_{rs})}{3}$, for $r, s = 1, 2, \ldots, n$.
where $\delta_{rs} = (POS_{rs}, NEU_{rs}, NEG_{rs})$ is the PF-edge between the nodes C_r and C_s for $r, s = 1, 2, \ldots, n$. Here, $\delta_{rs=} = 1$ and $\delta_{rs} = \delta_{sr}$ if $r = s$.

Step 2: Find the overall attribute of A_p by
$\widetilde{A_p} = (\widetilde{POS}_p, \widetilde{NEU}_p, \widetilde{NEG}_p) = \frac{1}{3} \sum_{j=1}^{n} w_j (\sum_{q=1}^{n} \delta_{qs} b_{pq})$, where, $f_{qj} = (POS_{qj}, NEU_{qj}, NEG_{qj})$.

Step 3: Compute the score value of $\widetilde{A_p}$ by
$score(\widetilde{A_p}) = \frac{1}{2}[1 + \widetilde{POS}_p - 2\widetilde{NEU}_p - \widetilde{NEG}_p]$

Step 4: Select the best alternative after ranking the alternative A_p depending on $score(\widetilde{A_p})$.

Step 5: Stop.

3|MADM using IVPFG

$IVPS$ has a lot of applications in real life. The $IVPFG$ can efficiently discuss uncertain situations in daily life. So, we will extend the $MADMM$ introduced by Amanathulla *et al.* [11] to solve $MADM$ problems with inter-valued PF-domain. The method developed in this article is $MADM$ method using $IVPFG$.

At first we discuss the decision making problem. Let the collection of alternatives be $A = \{A_1, A_2, \ldots, A_m\}$ and the collection of attributes be $C = \{C_1, C_2, \ldots, C_n\}$. Also, let the weight vector of the attribute C_i, $(i = 1, 2, \ldots, n)$ be $w = (w_1, w_2, \ldots, w_n)$, where $w_i \geq 0$ for $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{i=n} w_i = 1$. If the decision maker supply a picture fuzzy value for the alternative A_p, $p = 1, 2, \ldots, m$ under the attribute C_q, $q = 1, 2, \ldots, n$ then that value can be characterized by an interval valued picture fuzzy number $\zeta_{rs} = ([POS^l_{rs}, NEU^u_{rs}], [NEU^l_{rs}, NEG^u_{rs}], [NEG^l_{rs}, POS^u_{rs}])$, for $r = 1, 2, \ldots, m$, $s = 1, 2, \ldots, n$. Also, let the decision matrix be $M = (\zeta_{rs})_{m\times n}$, where ζ_{rs} is an $IVPF$-element. Also, if an $IVPF$-relation exists between the attributes $C_p = ([POS^l_p, NEU^u_p], [NEU^l_p, NEG^u_p], [NEG^l_p, NEG^u_p])$,
$C_q = ([POS^l_q, POS^u_q], [NEU^l_q, NEU^u_q], [NEG^l_q, NEG^u_q])$, we express the picture fuzzy relation by
$\psi_{rs} = ([POS^l_{rs}, NEU^u_{rs}], [NEU^l_{rs}, NEG^u_{rs}], [NEG^l_{rs}, POS^u_{rs}])$, for $r = 1, 2, \ldots, m$, $s = 1, 2, \ldots, n$, where,
$POS^l_{rs} \leq POS^l_r \wedge POS^l_s$, $POS^u_{rs} \leq POS^u_r \wedge POS^u_s$, $NEU^l_{rs} \geq NEU^l_r \vee NEU^l_s$, $NEU^u_{rs} \geq NEU^u_r \vee NEU^u_s$, $NEG^l_{rs} \geq NEG^l_r \vee NEG^l_s$, $NEG^u_{rs} \geq NEG^u_r \vee NEG^u_s$, for any $r, s = 1, 2, \ldots, m$; otherwise $\psi_{rs} = ([0.0, 0.0], [0.0, 0.0], [0.0, 0.0])$. To develope the graph structure, we have proposed an algorithm to make the decision regarding selection of best choice with $IVPF$-domain.

Algorithm 2

Step 1: Compute the coefficient of influence between the attributes C_r and C_s by
$\delta_{rs} = \frac{(POS^l_{rs}+POS^u_{rs})+\{2-(NEU^l_{rs}+NEU^u_{rs})\}\{2-(NEG^l_{rs}+NEG^u_{rs})\}}{6}$,
where, $\psi_{rs} = ([POS^l_{rs}, NEU^u_{rs}], [NEU^l_{rs}, NEG^u_{rs}], [NEG^l_{rs}, POS^u_{rs}])$ denote interval valued picture fuzzy edge between the nodes C_r, C_s (r,s=1,2,...,n). We have $\delta_{rs} = 1$ and $\delta_{rs} = \delta_{sr}$, for $r = s$.

Step 2: Compute the overall attribute of A_p, $p = 1, 2, \ldots, m$ by
$\widetilde{A_p} = \frac{1}{3}\sum_{s=1}^{n} w_s(\sum_{r=1}^{n} \zeta_{pr}\delta_{rs})$,
where, $\zeta_{pr} = ([POS^l_{pr}, NEU^u_{pr}], [NEU^l_{pr}, NEG^u_{pr}], [NEG^l_{pr}, NEG^u_{pr}])$ is an interval valued picture fuzzy number.

Step 3: Obtain the the score value of $\widetilde{A_p}$, $p = 1, 2, \ldots, m$ by
$score(\widetilde{A_p}) = \frac{1}{4}[2 + \widetilde{POS^l_p} + \widetilde{POS^u_p} - 2\widetilde{NEU^l_p} - 2\widetilde{NEU^u_p} - \widetilde{NEG^l_p} - \widetilde{NEG^u_p}]$

Step 4: Rank all the alternatives A_p for $p = 1, 2, \ldots, m$ and then select the optimal alternative according to the score value.

Step 5: Stop.

A flowchart of the above algorithm is shown in Fig. 5.

4|Numerical Example

To prove the effectiveness and application of our algorithm we consider a $MADM$ problem taken from Zhao *et al.* [15] with some modification, so that the elements of the decision matrix are interval valued picture fuzzy numbers.

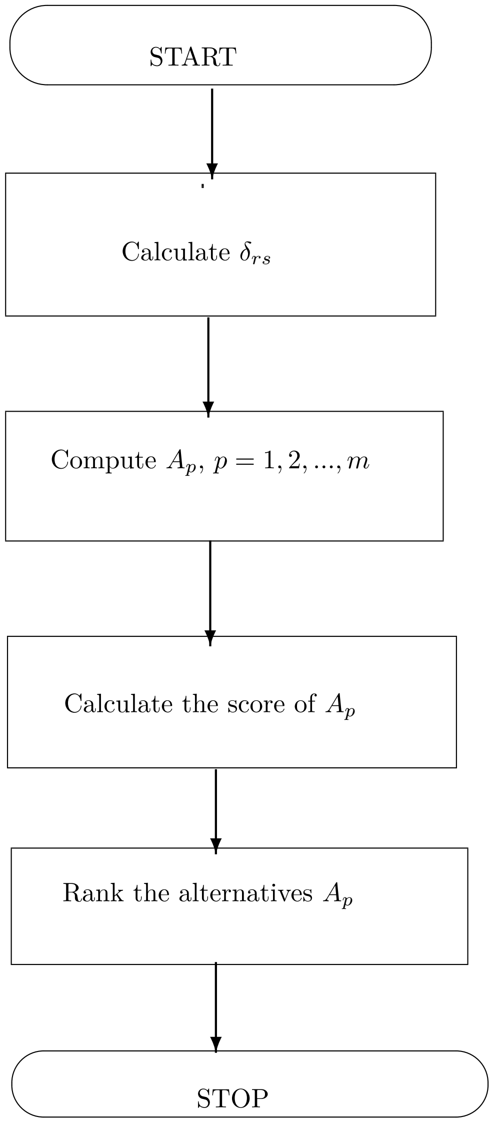

FIGURE 5. Flow chart of decision making algorithm

Example 1: A company wants to invest money in the best option. There are 4 possible alternatives in which the company invest their money:
A_1 : Car company
A_2 : Food company
A_3 : Computer company
A_4 : Arms company

The company take a decision according to the 3 attribute:
C_1 : Risk analysis
C_2 : Growth analysis
C_3 : Environmental impact analysis
The growth vector of the attribute is given by $w = (0.2, 0.25, 0.55)$.

Now under the three attribute, the decision maker has evaluated the four possible alternatives in the form of interval-valued picture fuzzy domain, consistent to attribute C_p, for $p = 1, 2, 3$ and the information evaluation

on the alternative A_q, $q = 1, 2, 3, 4$ under the three factors C_p, for $p = 1, 2, 3$ are shown in the following interval-valued picture fuzzy decision matrix

$$M = \begin{bmatrix} ([0.13, 0.17], [0.06, 0.1], [0.1, 0.13]) & ([0.13, 0.2], [0.03, 0.1], [0.06, 0.13] & ([0.13, 0.17], [0.06, 0.1], [0.23, 0.3]) \\ ([0.2, 0.23], [0.03, 0.06], [0.06, 0.1]) & ([0.2, 0.23], [0.03, 0.06], [0.06, 0.1]) & ([0.26, 0.3], [0.1, 0.17], [0.1, 0.2]) \\ ([0.1, 0.2], [0.06, 0.1], [0.1, 0.13]) & ([0.17, 0.2], [0.06, 0.1], [0.1, 0.13]) & ([0.23, 0.3], [0.06, 0.13], [0.13, 0.17]) \\ ([0.23, 0.27], [0.0, 0.03], [0.03, 0.06]) & ([0.2, 0.23], [0.03, 0.06], [0.03, 0.1]) & ([0.27, 0.3], [0.1, 0.13], [0.2, 0.23]) \end{bmatrix}$$

Also, we consider that the relation among the attributes C_1, C_2, C_3 can be expressed by a complete graph with three vertices $G = (V, E)$, where $V = \{C_1, C_2, C_3\}$ and $E = \{f_{12}, f_{13}, f_{23}\}$, see Fig. 6.

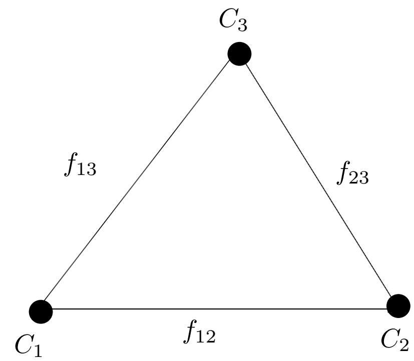

FIGURE 6. The graph relationship among the attributes

The relationship among the attributes are obtained from Step 2 of algorithm 2. Let us assume that the PF-edges denoting the relation among the attribute are discussed as followes:

$$\begin{aligned} f_{12} &= ([POS^l_{12}, POS^u_{12}], [NEU^i_{12}, NEU^u_{12}], [NEG^i_{12}, NEG^u_{12}]) \\ &= ([0.1, 0.13], [0.1, 0.17], [0.13, 0.17]) \end{aligned}$$

$$\begin{aligned} f_{13} &= ([POS^l_{13}, POS^u_{13}], [NEU^i_{13}, NEU^u_{13}], [NEG^i_{13}, NEG^u_{13}]) \\ &= ([0.06, 0.1], [0.13, 0.17], [0.17, 0.2]) \end{aligned}$$

$$\begin{aligned} f_{23} &= ([POS^l_{12}, POS^u_{12}], [NEU^i_{12}, NEU^u_{12}], [NEG^i_{12}, NEG^u_{12}]) \\ &= ([0.13, 0.2], [0.1, 0.13], [0.13, 0.17]) \end{aligned}$$

Notice that $G = (V, E)$ presents an $IVPFG$ according to the relation among attribute for every alternative. To find the best alternative we have to follow the steps below:

Step 1: The coefficient of influence are:

$$\begin{aligned} \delta_{12} &= \frac{(POS^l_{12} + POS^u_{12}) + \{2 - (NEU^l_{12} + NEU^u_{12})\}\{2 - (NEG^l_{12} + NEG^u_{12})\}}{6} \\ &= \frac{(0.1 + 0.13) + \{2 - (0.1 + 0.17)\}\{2 - (0.13 + 0.17)\}}{6} \\ &= \frac{0.23 + 1.73 \times 1.7}{6} \\ &= 0.5285 \end{aligned}$$

$$\begin{aligned} \delta_{13} &= \frac{(POS^l_{13} + POS^u_{13}) + \{2 - (NEU^l_{13} + NEU^u_{13})\}\{2 - (NEG^l_{13} + NEG^u_{13})\}}{6} \\ &= \frac{(0.06 + 0.1) + \{2 - (0.13 + 0.17)\}\{2 - (0.17 + 0.2)\}}{6} \\ &= \frac{0.16 + 1.7 \times 1.67}{6} \\ &= 0.48 \end{aligned}$$

$$\begin{aligned}
\delta_{23} &= \frac{(POS^l_{23}+POS^u_{23})+\{2-(NEU^l_{23}+NEU^u_{23})\}\{2-(NEG^l_{23}+NEG^u_{23})\}}{6}\\
&= \frac{(0.13+0.2)+\{2-(0.1+0.13)\}\{2-(0.13+0.17)\}}{6}\\
&= \frac{0.33+1.77\times 1.7}{6}\\
&= 0.5565
\end{aligned}$$

Therefore, $\delta_{11}=\delta_{22}=\delta_{33}=1$, $\delta_{12}=\delta_{21}=0.5285$, $\delta_{13}=\delta_{31}=0.48$, $\delta_{23}=\delta_{32}=0.5565$.

Step 2: We have

$$\begin{aligned}
\widetilde{A_1} &= \frac{1}{3}\sum_{s=1}^{3} w_s(\sum_{r=1}^{3}\zeta_{1r}\delta_{rs})\\
&= \frac{1}{3}[w_1\times(\zeta_{11}\delta_{11}+\zeta_{12}\delta_{21}+\zeta_{13}\delta_{31})+w_2\times(\zeta_{11}\delta_{12}+\zeta_{12}\delta_{22}+\zeta_{13}\delta_{32})\\
&+ w_3\times(\zeta_{11}\delta_{13}+\zeta_{12}\delta_{23}+\zeta_{13}\delta_{33})]\\
&= \frac{1}{3}[0.2\times\{1\times([0.13,0.17],[0.06,0.1],[0.1,0.13])+0.5285\times([0.13,0.2],[0.03,0.1],[0.06,0.13])\\
&+ 0.48\times([0.13,0.17],[0.06,0.1],[0.23,0.3])\}+0.25\times\{0.5285\times([0.13,0.17],[0.06,0.1],[0.1,0.13])\\
&+ 1\times([0.13,0.2],[0.03,0.1],[0.06,0.13])+0.5565\times([0.13,0.17],[0.06,0.1],[0.23,0.3])\}\\
&+ 0.55\times\{0.48\times([0.13,0.17],[0.06,0.1],[0.1,0.13])+0.5565\times([0.13,0.2],[0.03,0.1],[0.06,0.13])\\
&+ 1\times([0.13,0.17],[0.06,0.1],[0.23,0.3])\}]\\
&= \frac{1}{3}[0.2\times([0.2611,0.3573],[0.0567,0.1209],[0.1621,0.2387])\\
&+ 0.25\times([0.2711,0.3845],[0.0951,0.2085],[0.2488,0.3657])\\
&+ 0.55\times([0.2647,0.3629],[0.1055,0.2037],[0.3258,0.4347])]\\
&= ([0.0885,0.1162],[0.0310,0.0628],[0.0913,0.1261])
\end{aligned}$$

$$\begin{aligned}
\widetilde{A_2} &= \frac{1}{3}\sum_{s=1}^{3} w_s(\sum_{r=1}^{3}\zeta_{2r}\delta_{rs})\\
&= \frac{1}{3}[w_1\times(\zeta_{21}\delta_{11}+\zeta_{22}\delta_{21}+\zeta_{23}\delta_{31})+w_2\times(\zeta_{21}\delta_{12}+\zeta_{22}\delta_{22}+\zeta_{23}\delta_{32})\\
&+ w_3\times(\zeta_{21}\delta_{13}+\zeta_{22}\delta_{23}+\zeta_{23}\delta_{33})]\\
&= \frac{1}{3}[0.2\times\{1\times([0.2,0.23],[0.03,0.06],[0.06,0.1])+0.5285\times((([0.2,0.23],[0.03,0.06],[0.06,0.1])\\
&+ 0.48\times([0.26,0.3],[0.1,0.17],[0.1,0.2])\}+0.25\times\{0.5285\times([0.2,0.23],[0.03,0.06],[0.06,0.1])\\
&+ 1\times([0.2,0.23],[0.03,0.06],[0.06,0.1])+0.5565\times([0.26,0.3],[0.1,0.17],[0.1,0.2])\}\\
&+ 0.55\times\{0.48\times([0.2,0.23],[0.03,0.06],[0.06,0.1])+0.5565\times([0.2,0.23],[0.03,0.06],[0.06,0.1])\\
&+ 1\times([0.26,0.3],[0.1,0.17],[0.1,0.2])\}]\\
&= \frac{1}{3}[0.2\times([0.4305,0.4956],[0.0939,0.1733],[0.1397,0.2489])\\
&+ 0.25\times([0.4504,0.5185],[0.1015,0.1863],[0.1474,0.2642])\\
&+ 0.55\times([0.4813,0.5384],[0.1311,0.2322],[0.1622,0.3037])]\\
&= ([0.1506,0.1750],[0.0388,0.0696],[0.0513,0.0943])
\end{aligned}$$

$$
\begin{aligned}
\widetilde{A_3} &= \frac{1}{3}\sum_{s=1}^{3} w_s(\sum_{r=1}^{3}\zeta_{3r}\delta_{rs}) \\
&= \frac{1}{3}[w_1 \times (\zeta_{31}\delta_{11} + \zeta_{32}\delta_{21} + \zeta_{33}\delta_{31}) + w_2 \times (\zeta_{31}\delta_{12} + \zeta_{32}\delta_{22} + \zeta_{33}\delta_{32}) \\
&+ w_3 \times (\zeta_{31}\delta_{13} + \zeta_{32}\delta_{23} + \zeta_{33}\delta_{33})] \\
&= \frac{1}{3}[0.2 \times \{1 \times ([0.1, 0.2], [0.06, 0.1], [0.1, 0.13]) + 0.5285 \times ([0.17, 0.2], [0.06, 0.1], [0.1, 0.13]) \\
&+ 0.48 \times ([0.23, 0.3], [0.06, 0.13], [0.13, 0.17])\} + 0.25 \times \{0.5285 \times ([0.1, 0.2], [0.06, 0.1], [0.1, 0.1]) \\
&+ 1 \times ([0.17, 0.2], [0.06, 0.1], [0.1, 0.13]) + 0.5565 \times ([0.23, 0.3], [0.06, 0.13], [0.13, 0.17])\} \\
&+ 0.55 \times \{0.48 \times ([0.1, 0.2], [0.06, 0.1], [0.1, 0.13]) + 0.5565 \times ([0.17, 0.2], [0.06, 0.1], [0.1, 0.13]) \\
&+ 1 \times ([0.23, 0.3], [0.06, 0.13], [0.13, 0.17])\}] \\
&= \frac{1}{3}[0.2 \times ([0.3002, 0.4497], [0.1205, 0.2153], [0.2153, 0.2803]) \\
&+ 0.25 \times ([0.3508, 0.4727], [0.1251, 0.2252], [0.2252, 0.2933]) \\
&+ 0.55 \times ([0.3726, 0.5073], [0.1834, 0.2337], [0.2337, 0.3047])] \\
&= ([0.1176, 0.1624], [0.0521, 0.0760], [0.0760, 0.0990])
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{A_4} &= \frac{1}{3}\sum_{s=1}^{3} w_s(\sum_{r=1}^{3}\zeta_{4r}\delta_{rs}) \\
&= \frac{1}{3}[w_1 \times (\zeta_{41}\delta_{11} + \zeta_{42}\delta_{21} + \zeta_{43}\delta_{31}) + w_2 \times (\zeta_{41}\delta_{12} + \zeta_{42}\delta_{22} + \zeta_{43}\delta_{32}) \\
&+ w_3 \times (\zeta_{41}\delta_{13} + \zeta_{42}\delta_{23} + \zeta_{43}\delta_{33})] \\
&= \frac{1}{3}[0.2 \times \{1 \times ([0.23, 0.27], [0.0, 0.01], [0.03, 0.06]) + 0.5285 \times ([0.2, 0.23], [0.03, 0.06], [0.03, 0.1]) \\
&+ 0.48 \times ([0.27, 0.3], [0.1, 0.13], [0.2, 0.23])\} + 0.25 \times \{0.5285 \times ([0.23, 0.27], [0.0, 0.03], [0.03, 0.06]) \\
&+ 1 \times ([0.2, 0.23], [0.03, 0.06], [0.03, 0.1]) + 0.5565 \times ([0.27, 0.3], [0.1, 0.13], [0.2, 0.23])\} \\
&+ 0.55 \times \{0.48 \times ([0.23, 0.27], [0.0, 0.03], [0.03, 0.06]) + 0.5565 \times ([0.2, 0.23], [0.03, 0.06], [0.03, 0.1]) \\
&+ 1 \times ([0.27, 0.3], [0.1, 0.13], [0.2, 0.23])\}] \\
&= \frac{1}{3}[0.2 \times ([0.4653, 0.5356], [0.0639, 0.1241], [0.1419, 0.2233]) \\
&+ 0.25 \times ([0.4718, 0.5396], [0.0857, 0.1482], [0.1572, 0.2597]) \\
&+ 0.55 \times ([0.4917, 0.5576], [0.1167, 0.1778], [0.2311, 0.3145])] \\
&= ([0.1605, 0.1829], [0.0328, 0.0532], [0.0649, 0.0942])
\end{aligned}
$$

Step 3: Now the score value of A_1, A_2, A_3 and A_4 are calculated as follows:

$$
\begin{aligned}
score(\widetilde{A_1}) &= \frac{1}{4}[2 + \widetilde{POS_1^l} + \widetilde{POS_1^u} - 2\widetilde{NEU_1^l} - 2\widetilde{NEU_1^u} - \widetilde{NEG_1^l} - \widetilde{NEG_1^u}] \\
&= \frac{1}{4}[2 + 0.0885 + 0.1162 - 2 \times 0.0310 - 2 \times 0.0628 - 0.0913 - 00.1261] \\
&= 0.4499
\end{aligned}
$$

$$
\begin{aligned}
score(\widetilde{A_2}) &= \frac{1}{4}[2 + \widetilde{POS_2^l} + \widetilde{POS_2^u} - 2\widetilde{NEU_2^l} - 2\widetilde{NEU_2^u} - \widetilde{NEG_2^l} - \widetilde{NEG_2^u}] \\
&= \frac{1}{4}[2 + 0.1506 + 0.1750 - 2 \times 0.0388 - 2 \times 0.0696 - 0.0513 - 0.0943] \\
&= 0.4908
\end{aligned}
$$

	Include membership values	Include non-membership values	Include neutral membership values	Flexibility permits in membership values	Flexibility permits in non-membership values	Flexibility permits in neutral membership values
$MADM$ on FG	✓	×	×	×	×	×
$MADM$ on $IVFG$	✓	×	×	✓	×	×
$MADM$ on IFG	✓	✓	×	×	×	×
$MADM$ on $IVIFG$	✓	✓	×	✓	✓	×
Operation on PFG	✓	✓	✓	×	×	×
$MADM$ on $IVIFG$	✓	✓	✓	✓	✓	✓

TABLE 3. Comparative study table

$$\begin{aligned}
score(\widetilde{A_3}) &= \frac{1}{4}[2+\widetilde{POS_3^l}+\widetilde{POS_3^u}-2\widetilde{NEU_3^l}-2\widetilde{NEU_3^u}-\widetilde{NEG_3^l}-\widetilde{NEG_3^u}] \\
&= \frac{1}{4}[2+0.1176+0.1624-2\times 0.0521-2\times 0.0760-0.0760-0.0990] \\
&= 0.4622 \\
score(\widetilde{A_4}) &= \frac{1}{4}[2+\widetilde{POS_4^l}+\widetilde{POS_4^u}-2\widetilde{NEU_4^l}-2\widetilde{NEU_4^u}-\widetilde{NEG_4^l}-\widetilde{NEG_4^u}] \\
&= \frac{1}{4}[2+0.1605+0.1829-2\times 0.0328-2\times 0.0532-0.0649-0.0942] \\
&= 0.5031
\end{aligned}$$

Step 4: Since, $score(\widetilde{A_4}) > score(\widetilde{A_2}) > score(\widetilde{A_3}) > score(\widetilde{A_1})$, so,the rank of the alternatives is $A_4 > A_2 > A_3 > A_1$. Hence, the best choice of the alternative is A_4. Therefore, the company invest the money in an arms company.

Comparative Analysis. The ranking obtained aligns with the findings of Zaho *et al.* [15], who tackled a similar problem as outlined in [15]. As a result, the aforementioned investigation underscores the viability of this approach within the interval-valued picture fuzzy domain. Furthermore, it becomes evident that this method offers unique insights compared to other methods within picture fuzzy environments, rendering it a valuable and versatile tool. PFG based decision making model are more general than fuzzy graph, intuitionistic fuzzy graph models. In PFG the positive, neutral and negative membership values of nodes and edges are take as a fixed number in $[0,1]$. But, it is very difficult task to fix a number for the membership values of nodes and edges. $IVPFG$ gives the opportunities for flexibality of membership values. In $IVPFG$ the positive, neutral and negative membership values are taken as an interval in $[0,1]$. So, the results regarding decision making problem in $IVPFG$ is more general than interval valued fuzzy graphs, interval valued intuitionistic fuzzy graphs. Therefore, our proposed $MADM$ problem using $IVPFG$ are more general and flexible. The generality and flexibility of the proposed method is shown in Table 3.

5|Conclusion and future work

This article introduced $IVPFG$ and some important definitions and properties of $IVPFG$s. Also, we developed a decision-making algorithm to solve complicated $MADM$ problem using $IVPFG$. Besides this, a numerical

example has been given to discuss the application of the proposed decision-making algorithm. The limitations of the proposed methods are the method can become computationally intensive as the number of attributes and their interrelations increase and the approach may offer little benefit in situations where attributes are well-defined and deterministic. In the future, the new researcher can apply this technique to solve various types of decision-making problem in different fuzzy environments like, fermatean fuzzy graphs, quasirung fuzzy graphs, etc.

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Author Contribution

J. Khatun: Definitions and theorems deduction. S. Amanathulla: Algorithm writing, writing and editing of the article. All authors have read and agreed to the published version of the manuscript.

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Data Availability

No external data were used to support this study.

Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings. Funders played no role in the study's design, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results.

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